Math 10A HW2 Solutions

(1) True !

(2) We first find the slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-2}{5-1} = \frac{5}{4}$.

Then we get the equation $y - 2 = \frac{5}{4}(x-1)$

or equivalently $y = \frac{5}{4}x - \frac{5}{4} + 2 = \frac{5}{4}x + \frac{3}{4}$.

(3) (a) $d/dx\,(3) = 0$

(b) $d/dx\,(x^4) = 4x^3$

(c) $d/dx\,(3x^2+1) = 6x$

(d) $d/dx\,(x^{-2}) = -2x^{-3} = -2/x^3$

(4) Notice that $\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \frac{h}{h} = 1$

whereas $\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \frac{-h}{h} = -1$

so the derivative of f at 0 DOES NOT EXIST!

(5) (a) $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \frac{d}{dx}(x^2) = 2x$

(b) $\lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h} = \frac{d}{dx}(\tan x) = \sec^2(x)$

(c) $\lim_{x\to \pi/3} \frac{\sin(x) - \sin(\pi/3)}{x - \pi/3} = \frac{d}{dx}(\sin x)\Big|_{x=\pi/3}$

$= \cos(x)\Big|_{x=\pi/3} = \cos(\pi/3) = 1/2$

(d) $\lim_{x\to 1} \frac{\ln(x)}{x-1} = \lim_{x\to 1} \frac{\ln(x) - \ln(1)}{x-1}$

$= \frac{d}{dx}(\ln x)\Big|_{x=1} = \frac{1}{x}\Big|_{x=1} = 1$

(e) $\lim_{x\to 0} \frac{\sin(x)}{x} = \lim_{x\to 0} \frac{\sin(x) - \sin(0)}{x - 0}$

$= \frac{d}{dx}(\sin x)\Big|_{x=0} = \cos x\Big|_{x=0} = \cos(0) = 1$

(6) True, by definition

(7) False, let $f(x) = 1/x$ so $f'(x) = -1/x^2$ with $a=1$. Then $f'(1) = -1$ but $f(1) = 1$.

(8) False, let $f(x) = x^2$ then $f'(0) = 0$ but f is not constant function.

(9) True, notice that $(f+g)'(a) = f'(a) + g'(a)$ with each term $> 0 \Rightarrow$ sum > 0.

(10) False, $\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -[f(x)]^{-2} \cdot f'(x)$ by chain rule.

(11) True, let $f(x) = |x-1|$.

(12) (a) $f(x) = \frac{1}{2+\cos x}$, $a = \pi$

Then $f'(x) = \frac{d}{dx}[(2+\cos x)^{-1}]$

$= -1(2+\cos x)^{-2} \cdot (-\sin x)$

$= \frac{\sin x}{(2+\cos x)^2}$

$\Rightarrow f'(\pi) = \frac{\sin \pi}{(2+\cos \pi)^2} = 0$; $f(\pi) = \frac{1}{2+\cos \pi} = 1$

So equation of tangent line is

$y - 1 = 0(x-\pi) \Rightarrow y = 1$.

(b) $f(x) = e^x - 1$, $a = \ln(3)$

Then $f'(x) = e^x$ so $f'(\ln 3) = 3$

where $f(\ln 3) = 3 - 1 = 2$.

So equation of tangent line is

$y - 2 = 3(x - \ln 3)$.

(c) $f(x) = (2x)^{-1}$, $a = -4$

Then $f'(x) = -(2x)^{-2} \cdot 2 = -2/4x^2 = \frac{-1}{2x^2}$

where $f(-4) = (-8)^{-1} = {}^{-1}/8$.

So equation of tangent line is

$y + {}^{1}/8 = \left(\frac{-1}{2 \cdot 16}\right)(x+4)$

$\Rightarrow y + {}^{1}/8 = {}^{-1}/32\,(x+4)$.

(d) $f(x) = x^{1/2}$, $a = 9$

Then $f'(x) = {}^{1}/2\, x^{-1/2}$ so $f'(9) = \frac{1}{2 \cdot 3} = \frac{1}{6}$

where $f(9) = 3$.

So equation of tangent line is

$y - 3 = {}^{1}/6\,(x-9)$.

(13) (a) $d/dx\,(\cos x) = -\sin x$

(b) $d/dx\,(x \ln x) = x \cdot {}^{1}/x + \ln(x) \cdot 1$

$= 1 + \ln(x)$

(c) $d/dx\,((1-x)^{-1}) = -1\,(1-x)^{-2}\,(-1)$

$= {}^{1}/(1-x)^2$

(d) $d/dx \left(\frac{3x^3+1}{4x^2-2}\right) = \frac{(4x^3-2)(9x^2) - (3x^3+1)(8x)}{(4x^2-2)^2}$

(e) $\frac{d}{dx} \cos(\tan(3x)) = -\sin(\tan(3x)) \cdot \sec^2(3x) \cdot 3$

$= -3 \sin(\tan 3x) \sec^2(3x)$

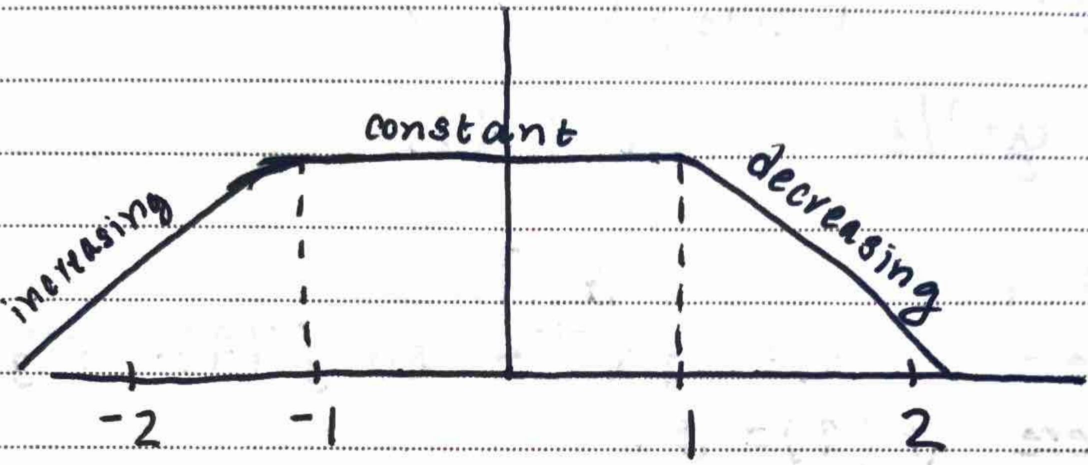
(f) $\frac{d}{dx} \tan(x^2) = \sec^2(x^2) \cdot 2x$

(g) $\frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$

(h) $\frac{d}{dx}\left(\sin(x)\cos x\right) = \sin(x)\cdot(-\sin x) + \cos(x)\cdot(\cos x)$

$= -\sin^2(x) + \cos^2 x$

(14)



(15) The derivative is undefined at $x=-2,-1,0,1$ ("corners").
We estimate that $f'(-1.5) < f'(-0.5) < f'(2)$.

(16) False, consider absolute value function

(17) False, let $f(x) = |x-1|$ so f differentiable on $(0,1)$ and $(1,2)$ but NOT at 1.

(18) False, $(fg)'(x) = f(x)g'(x) + g(x)f'(x)$.